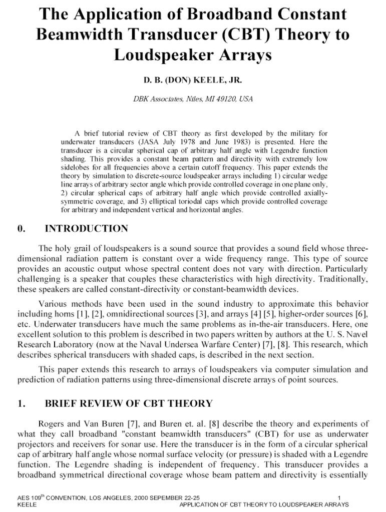
Audio Engineering Society and Acoustical Society CBT Technical Papers

Keele CBT Paper #1 (2000)



The Application of Broadband Constant Beamwidth Transducer (CBT) Theory to Loudspeaker Arrays

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A brief tutorial review of CBT theory as first developed by the military for underwater transducers (JASA July 1978 and June 1983) is presented. Here the transducer is a circular spherical cap of arbitrary half angle with Legendre function shading. This provides a constant beam pattern and directivity with extremely low sidelobes for all frequencies above a certain cutoff frequency. This paper extends the theory by simulation to discrete-source loudspeaker arrays including 1) circular wedge line arrays of arbitrary sector angle which provide controlled coverage in one plane only, 2) circular spherical caps of arbitrary half angle which provide controlled axially-symmetric coverage, and 3) elliptical toriodal caps which provide controlled coverage for arbitrary and independent vertical and horizontal angles.

0. INTRODUCTION

The holy grail of loudspeakers is a sound source that provides a sound field whose three-dimensional radiation pattern is constant over a wide frequency range. This type of source provides an acoustic output whose spectral content does not vary with direction. Particularly challenging is a speaker that couples these characteristics with high directivity. Traditionally, these speakers are called constant-directivity or constant-beamwidth devices.

Various methods have been used in the sound industry to approximate this behavior including horns [1], [2], omnidirectional sources [3], and arrays [4] [5], higher-order sources [6], etc. Underwater transducers have much the same problems as in-the-air transducers. Here, one excellent solution to this problem is described in two papers written by authors at the U. S. Navel Research Laboratory (now at the Naval Undersea Warfare Center) [7], [8]. This research, which describes spherical transducers with shaded caps, is described in the next section.

This paper extends this research to arrays of loudspeakers via computer simulation and prediction of radiation patterns using three-dimensional discrete arrays of point sources.

1. BRIEF REVIEW OF CBT THEORY

Rogers and Van Buren [7], and Buren et. al. [8] describe the theory and experiments of what they call broadband "constant beamwidth transducers" (CBT) for use as underwater projectors and receivers for sonar use. Here the transducer is in the form of a circular spherical cap of arbitrary half angle whose normal surface velocity (or pressure) is shaded with a Legendre function. The Legendre shading is independent of frequency. This transducer provides a broadband symmetrical directional coverage whose beam pattern and directivity is essentially

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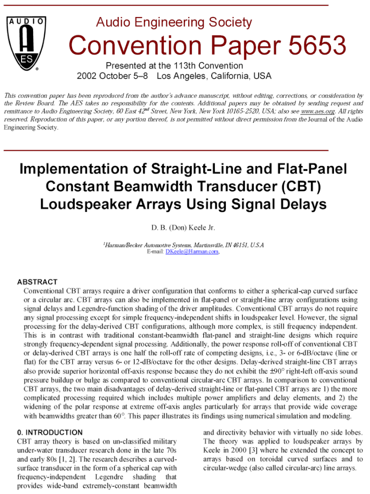
Keele CBT Paper #2 (2002)



Audio Engineering Society

Convention Paper 5653

Presented at the 113th Convention
2002 October 5–8 Los Angeles, California, USA

Implementation of Straight-Line and Flat-Panel Constant Beamwidth Transducer (CBT) Loudspeaker Arrays Using Signal Delays

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ABSTRACT

Conventional CBT arrays require a driver configuration that conforms to either a spherical-cap curved surface or a circular arc. CBT arrays can also be implemented in flat-panel or straight-line array configurations using signal delays and Legendre-function shading of the driver amplitudes. Conventional CBT arrays do not require any signal processing except for simple frequency-independent shifts in loudspeaker level. However, the signal processing for the delay-derived CBT configurations, although more complex, is still frequency independent. This is in contrast with traditional constant-beamwidth flat-panel and straight-line designs which require strongly frequency-dependent signal processing. Additionally, the power response roll-off of conventional CBT or delay-derived CBT arrays is one half the roll-off rate of competing designs, i.e., 3- or 6-dB/octave (line or flat) for the CBT array versus 6- or 12-dB/octave for the other designs. Delay-derived straight-line CBT arrays also provide superior horizontal off-axis response because they do not exhibit the ±90° right-left off-axis sound pressure buildup or bulge as compared to conventional circular-arc CBT arrays. In comparison to conventional CBT arrays, the two main disadvantages of delay-derived straight-line or flat-panel CBT arrays are 1) the more complicated processing required which includes multiple power amplifiers and delay elements, and 2) the widening of the polar response at extreme off-axis angles particularly for arrays that provide wide coverage with beamwidths greater than 60°. This paper illustrates its findings using numerical simulation and modeling.

0. INTRODUCTION

CBT array theory is based on un-classified military under-water transducer research done in the late 70s and early 80s [1, 2]. The research describes a curved-surface transducer in the form of a spherical cap with frequency-independent Legendre shading that provides wide-band extremely-constant beamwidth and directivity behavior with virtually no side lobes. The theory was applied to loudspeaker arrays by Keele in 2000 [3] where he extended the concept to arrays based on toroidal curved surfaces and to circular-wedge (also called circular-arc) line arrays.

Keele CBT Paper #3 (2003)

PAPERS

Full-Sphere Sound Field of Constant-Beamwidth Transducer (CBT) Loudspeaker Line Arrays*

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The full-sphere sound radiation pattern of the constant-beamwidth transducer circular-wedge curved-line loudspeaker array exhibits a three-dimensional petal or eye-shaped sound radiation pattern that stays surprisingly uniform with frequency. Oriented vertically, it not only exhibits the expected uniform control of vertical coverage, but also provides significant coverage control horizontally. The horizontal control is provided by a vertical coverage that decreases smoothly as a function of the horizontal off-axis angle and reaches a minimum at right angles to the primary listening axis. This is in contrast to a straight-line array, which exhibits a three-dimensional sound field that is axially symmetric about its vertical axis and exhibits only minimal directivity in the horizontal plane due to the inherent directional characteristics of each of the sources that make up the array.

0 INTRODUCTION

Constant-beamwidth transducer (CBT) array theory is based on unclassified military underwater transducer research done in the late 1970s and early 1980s [1][2]. The research describes a curved-surface transducer in the form of a spherical cap with frequency-independent Legendre shading that provides wide-band extremely constant beamwidth and directivity behavior with virtually no sidelobes. In 2000 Keele applied the theory to loudspeaker arrays [3] where he extended the concept to arrays based on toroid-shaped curved surfaces and to circular-arc line arrays. He also extended the concept to straight-line and flat-panel CBT arrays with the use of signal delays [4]. The Appendix gives a brief review of CBT theory.

Traditionally line arrays are thought to provide directional control in one plane only. This is quite true for straight-line arrays, but not for curved-line ones. In addition to the expected coverage control in the plane of the curved array, the curvature of the line array also provides directional control at off-axis angles in the opposite planes. For the typical vertically oriented curved-line array this means that the array provides not only the expected directional control vertically, but also horizontally.

The horizontal directional control provided by the curvature of the line array is primarily exhibited by a narrowing vertical coverage as a function of the horizontal off-axis angle and an increasing level as the listener location proceeds off axis horizontally. In front of the curved-line array, along the primary listening axis, the curvature of the array places sources at different distances from the listening location and provides the primary directional control mechanism. However, for off-axis horizontal locations, the curvature of the array is less and less evident as one proceeds to greater off-axis horizontal angles. At ±90° off axis horizontally, the curvature is entirely nullified and all sources are essentially equidistant from far-field listening or observation locations. At these locations the array appears essentially as a straight-line array, providing both maximum level and the narrowest possible vertical coverage. Interestingly the curved-line array thus provides its maximum intensity at right angles to its primary listening axis.

Further analysis of the CBT circular-wedge line array reveals that its vertical coverage gets narrower and narrower as one proceeds off axis horizontally. The narrowing of vertical coverage is found to follow the cosine of the off-axis horizontal angle. This means that the vertical coverage of the array narrows smoothly from its designed on-axis value and reaches a minimum at right angles horizontal to the primary listening axis.

The full-sphere sound field of the CBT curved-line array reveals a characteristic three-dimensional petal- or eye-shaped sound radiation pattern, which stays surprisingly uniform with frequency. This provides very uniform directivity and beamwidth control above a certain frequency set by the size of the array and its designed vertical coverage. Although the horizontal ±90° off-axis intensity is higher than the primary designed listening axis, the effect is essentially negated because the vertical coverage

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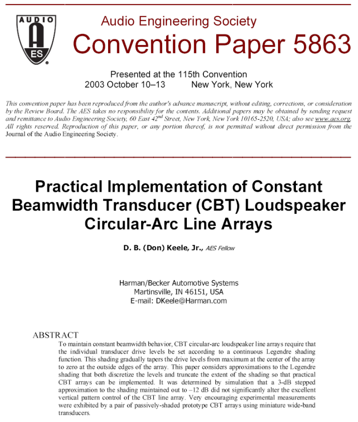
Keele CBT Paper #4 (2003)



Audio Engineering Society

Convention Paper 5863

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2003 October 10–13 New York, New York

Practical Implementation of Constant Beamwidth Transducer (CBT) Loudspeaker Circular-Arc Line Arrays

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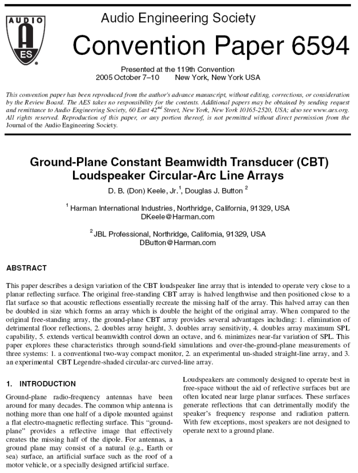
ABSTRACT

To maintain constant beamwidth behavior, CBT circular-arc loudspeaker line arrays require that the individual transducer drive levels be set according to a continuous Legendre shading function. This shading gradually tapers the drive levels from maximum at the center of the array to zero at the outside edges of the array. This paper considers approximations to the Legendre shading that both discretize the levels and truncate the extent of the shading so that practical CBT arrays can be implemented. It was determined by simulation that a 3-dB stepped approximation to the shading maintained out to –12 dB did not significantly alter the excellent vertical pattern control of the CBT line array. Very encouraging experimental measurements were exhibited by a pair of passively-shaded prototype CBT arrays using miniature wide-band transducers.

Keele CBT Paper #5 (2005)



Audio Engineering Society

Convention Paper 6594

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2005 October 7–10 New York, New York USA

Ground-Plane Constant Beamwidth Transducer (CBT) Loudspeaker Circular-Arc Line Arrays

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ABSTRACT

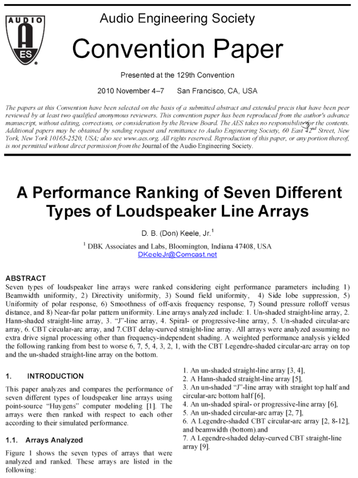
This paper describes a design variation of the CBT loudspeaker line array that is intended to operate very close to a planar reflecting surface. The original free-standing CBT array is halved lengthwise and then positioned close to a flat surface so that acoustic reflections essentially recreate the missing half of the array. This halved array can then be doubled in size which forms an array which is double the height of the original array. When compared to the original free-standing array, the ground-plane CBT array provides several advantages including: 1. elimination of detrimental floor reflections, 2. doubles array height, 3. doubles array sensitivity, 4. doubles array maximum SPL capability, 5. extends vertical beamwidth control down an octave, and 6. minimizes near-far variation of SPL. This paper explores these characteristics through sound-field simulations and over-the-ground-plane measurements of three systems: 1. a conventional two-way compact monitor, 2. an experimental un-shaded straight-line array, and 3. an experimental CBT Legendre-shaded circular-arc curved-line array.

1. INTRODUCTION

Ground-plane radio-frequency antennas have been around for many decades. The common whip antenna is nothing more than one half of a dipole mounted against a flat electro-magnetic reflecting surface. This "ground-plane" provides a reflective image that effectively creates the missing half of the dipole. For antennas, a ground plane may consist of a natural (e.g., Earth or sea) surface, an artificial surface such as the roof of a motor vehicle, or a specially designed artificial surface.

Loudspeakers are commonly designed to operate best in free-space without the aid of reflective surfaces but are often located near large planar surfaces. These surfaces generate reflections that can detrimentally modify the speaker's frequency response and radiation pattern. With few exceptions, most speakers are not designed to operate next to a ground plane.

Keele CBT Paper #6 (2010)



Audio Engineering Society

Convention Paper

Presented at the 129th Convention

2010 November 4–7 San Francisco, CA, USA

A Performance Ranking of Seven Different Types of Loudspeaker Line Arrays

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ABSTRACT

Seven types of loudspeaker line arrays were ranked considering eight performance parameters including 1) Beamwidth uniformity, 2) Directivity uniformity, 3) Sound field uniformity, 4) Side lobe suppression, 5) Uniformity of polar response, 6) Smoothness of off-axis frequency response, 7) Sound pressure rolloff versus distance, and 8) Near-far polar pattern uniformity. Line arrays analyzed include: 1. Un-shaded straight-line array, 2. Hann-shaded straight-line array, 3. "J"-line array, 4. Spiral- or progressive-line array, 5. Un-shaded circular-arc array, 6. CBT circular-arc array, and 7.CBT delay-curved straight-line array. All arrays were analyzed assuming no extra drive signal processing other than frequency-independent shading. A weighted performance analysis yielded the following ranking from best to worse 6, 7, 5, 4, 3, 2, 1, with the CBT Legendre-shaded circular-arc array on top and the un-shaded straight-line array on the bottom.

1. INTRODUCTION

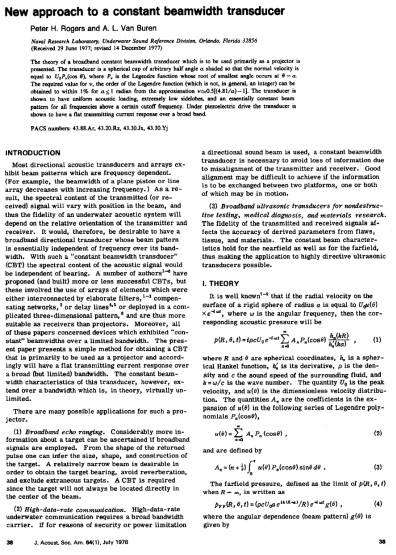
This paper analyzes and compares the performance of seven different types of loudspeaker line arrays using point-source "Huygens" computer modeling [1]. The arrays were then ranked with respect to each other according to their simulated performance.

1.1. Arrays Analyzed

Figure 1 shows the seven types of arrays that were analyzed and ranked. These arrays are listed in the following:

1. An un-shaded straight-line array [3, 4],
2. A Hann-shaded straight-line array [5],
3. An un-shaded "J"-line array with straight top half and circular-arc bottom half [6],
4. An un-shaded spiral- or progressive-line array [6],
5. An un-shaded circular-arc array [2, 7],
6. A Legendre-shaded CBT circular-arc array [2, 8-12], and beamwidth (bottom).and
7. A Legendre-shaded delay-curved CBT straight-line array [9].

U.S. Navy CBT Paper #1 (1978)



New approach to a constant beamwidth transducer

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(Received 29 June 1977; revised 14 December 1977)

The theory of a broadband constant beamwidth transducer which is to be used primarily as a projector is presented. The transducer is a spherical cap of arbitrary half angle α shaded so that the normal velocity is equal to $U_0P_\nu(\cos\theta)$, where P_ν is the Legendre function whose root of smallest angle occurs at $\theta=\alpha$. The required value for ν, the order of the Legendre function (which is not, in general, an integer) can be obtained to within 1% for $\alpha\le 1$ radian from the approximation $\nu\simeq 0.5[(4.81/\alpha)-1]$. The transducer is shown to have uniform acoustic loading, extremely low sidelobes, and an essentially constant beam pattern for all frequencies above a certain cutoff frequency. Under piezoelectric drive the transducer is shown to have a flat transmitting current response over a broad band.

PACS numbers: 43.88.Ar, 43.20.Rz, 43.30.Jx, 43.30.Yj

INTRODUCTION

Most directional acoustic transducers and arrays exhibit beam patterns which are frequency dependent. (For example, the beamwidth of a plane piston or line array decreases with increasing frequency.) As a result, the spectral content of the transmitted (or received) signal will vary with position in the beam, and thus the fidelity of an underwater acoustic system will depend on the relative orientation of the transmitter and receiver. It would, therefore, be desirable to have a broadband directional transducer whose beam pattern is essentially independent of frequency over its bandwidth. With such a "constant beamwidth transducer" (CBT) the spectral content of the acoustic signal would be independent of bearing. A number of authors[1–6] have proposed (and built) more or less successful CBTs, but these involved the use of arrays of elements which were either interconnected by elaborate filters,[1–3] compensating networks,[3] or delay lines[4,5] or deployed in a complicated three-dimensional pattern,[6] and are thus more suitable as receivers than projectors. Moreover, all of these papers concerned devices which exhibited "constant" beamwidths over a limited bandwidth. The present paper presents a simple method for obtaining a CBT that is primarily to be used as a projector and accordingly will have a flat transmitting current response over a broad (but limited) bandwidth. The constant beamwidth characteristics of this transducer, however, extend over a bandwidth which is, in theory, virtually unlimited.

There are many possible applications for such a projector.

(1) *Broadband echo ranging.* Considerably more information about a target can be ascertained if broadband signals are employed. From the shape of the returned pulse one can infer the size, shape, and construction of the target. A relatively narrow beam is desirable in order to obtain the target bearing, avoid reverberation, and exclude extraneous targets. A CBT is required since the target will not always be located directly in the center of the beam.

(2) *High-data-rate communication.* High-data-rate underwater communication requires a broad bandwidth carrier. If for reasons of security or power limitation a directional sound beam is used, a constant beamwidth transducer is necessary to avoid loss of information due to misalignment of the transmitter and receiver. Good alignment may be difficult to achieve if the information is to be exchanged between two platforms, one or both of which may be in motion.

(3) *Broadband ultrasonic transducers for nondestructive testing, medical diagnosis, and materials research.* The fidelity of the transmitted and received signals affects the accuracy of derived parameters from flaws, tissue, and materials. The constant beam characteristics hold for the nearfield as well as for the farfield, thus making the application to highly directive ultrasonic transducers possible.

I. THEORY

It is well known[7–9] that if the radial velocity on the surface of a rigid sphere of radius a is equal to $U_0u(\theta)\times e^{-i\omega t}$, where ω is the angular frequency, then the corresponding acoustic pressure will be

$$p(R,\theta,t)=i\rho cU_0e^{-i\omega t}\sum_{n=0}^{\infty}A_nP_n(\cos\theta)\frac{h_n(kR)}{h_n'(ka)}\,, \tag{1}$$

where R and θ are spherical coordinates, h_n is a spherical Hankel function, h_n' is its derivative, ρ is the density and c the sound speed of the surrounding fluid, and $k=\omega/c$ is the wave number. The quantity U_0 is the peak velocity, and $u(\theta)$ is the dimensionless velocity distribution. The quantities A_n are the coefficients in the expansion of $u(\theta)$ in the following series of Legendre polynomials $P_n(\cos\theta)$,

$$u(\theta)=\sum_{n=0}^{\infty}A_nP_n(\cos\theta)\,, \tag{2}$$

and are defined by

$$A_n=(n+\tfrac{1}{2})\int_0^{\pi}u(\theta)P_n(\cos\theta)\sin\theta\,d\theta\,. \tag{3}$$

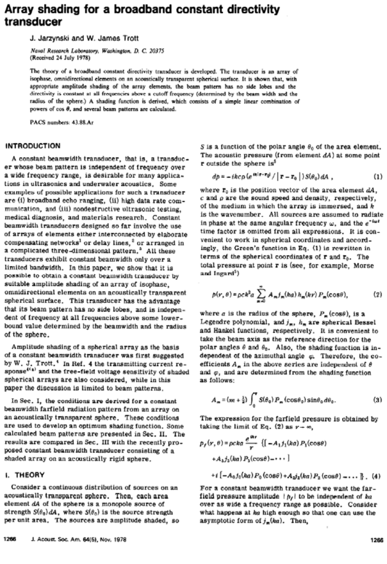
The farfield pressure, defined as the limit of $p(R,\theta,t)$ when $R\to\infty$, is written as

$$p_{FF}(R,\theta,t)=(\rho cU_0a\,e^{ik(R-a)}/R)\,e^{-i\omega t}g(\theta)\,, \tag{4}$$

where the angular dependence (beam pattern) $g(\theta)$ is given by

38 J. Acoust. Soc. Am. 64(1), July 1978 38

U.S. Navy CBT Paper #2 (1978)



Array shading for a broadband constant directivity transducer

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(Received 24 July 1978)

The theory of a broadband constant directivity transducer is developed. The transducer is an array of isophase, omnidirectional elements on an acoustically transparent spherical surface. It is shown that, with appropriate amplitude shading of the array elements, the beam pattern has no side lobes and the directivity is constant at all frequencies above a cutoff frequency (determined by the beam width and the radius of the sphere.) A shading function is derived, which consists of a simple linear combination of powers of cos θ, and several beam patterns are calculated.

PACS numbers: 43.88.Ar

INTRODUCTION

A constant beamwidth transducer, that is, a transducer whose beam pattern is independent of frequency over a wide frequency range, is desirable for many applications in ultrasonics and underwater acoustics. Some examples of possible applications for such a transducer are (i) broadband echo ranging, (ii) high data rate communication, and (iii) nondestructive ultrasonic testing, medical diagnosis, and materials research. Constant beamwidth transducers designed so far involve the use of arrays of elements either interconnected by elaborate compensating networks[1] or delay lines,[2] or arranged in a complicated three-dimensional pattern.[3] All these transducers exhibit constant beamwidth only over a limited bandwidth. In this paper, we show that it is possible to obtain a constant beamwidth transducer by suitable amplitude shading of an array of isophase, omnidirectional elements on an acoustically transparent spherical surface. This transducer has the advantage that its beam pattern has no side lobes, and is independent of frequency at all frequencies above some lower-bound value determined by the beamwidth and the radius of the sphere.

Amplitude shading of a spherical array as the basis of a constant beamwidth transducer was first suggested by W. J. Trott.[4] In Ref. 4 the transmitting current response[4(a)] and the free-field voltage sensitivity of shaded spherical arrays are also considered, while in this paper the discussion is limited to beam patterns.

In Sec. I, the conditions are derived for a constant beamwidth farfield radiation pattern from an array on an acoustically transparent sphere. These conditions are used to develop an optimum shading function. Some calculated beam patterns are presented in Sec. II. The results are compared in Sec. III with the recently proposed constant beamwidth transducer consisting of a shaded array on an acoustically rigid sphere.

I. THEORY

Consider a continuous distribution of sources on an acoustically transparent sphere. Then, each area element dA of the sphere is a monopole source of strength $S(\theta_0)\,dA$, where $S(\theta_0)$ is the source strength per unit area. The sources are amplitude shaded, so S is a function of the polar angle θ_0 of the area element. The acoustic pressure (from element dA) at some point $\mathbf{r}$ outside the sphere is[5]

$$dp=-ikc\rho\,(e^{ik|\mathbf{r}-\mathbf{r}_0|}/|\mathbf{r}-\mathbf{r}_0|)\,S(\theta_0)\,dA\,, \tag{1}$$

where $\mathbf{r}_0$ is the position vector of the area element dA, c and ρ are the sound speed and density, respectively, of the medium in which the array is immersed, and k is the wavenumber. All sources are assumed to radiate in phase at the same angular frequency ω, and the $e^{-i\omega t}$ time factor is omitted from all expressions. It is convenient to work in spherical coordinates and accordingly, the Green's function in Eq. (1) is rewritten in terms of the spherical coordinates of $\mathbf{r}$ and $\mathbf{r}_0$. The total pressure at point $\mathbf{r}$ is (see, for example, Morse and Ingard[5])

$$p(r,\theta)=\rho ck^2a\sum_{m=0}^{\infty}A_mj_m(ka)\,h_m(kr)\,P_m(\cos\theta), \tag{2}$$

where a is the radius of the sphere, $P_m(\cos\theta)$, is a Legendre polynomial, and j_m, h_m are spherical Bessel and Hankel functions, respectively. It is convenient to take the beam axis as the reference direction for the polar angles θ and θ_0. Also, the shading function is independent of the azimuthal angle φ. Therefore, the coefficients A_m in the above series are independent of θ and φ, and are determined from the shading function as follows:

$$A_m=(m+\tfrac{1}{2})\int_0^{\pi}S(\theta_0)\,P_m(\cos\theta_0)\sin\theta_0\,d\theta_0. \tag{3}$$

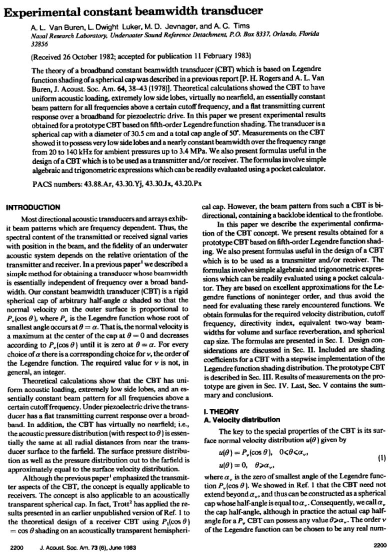
The expression for the farfield pressure is obtained by taking the limit of Eq. (2) as $r\to\infty$,

$$\begin{aligned}p_f(r,\theta)=\rho cka\,\frac{e^{ikr}}{r}\,\{&[-A_1j_1(ka)P_1(\cos\theta)\\&+A_3j_3(ka)P_3(\cos\theta)-\cdots]\\&+i[-A_0j_0(ka)P_0(\cos\theta)+A_2j_2(ka)P_2(\cos\theta)-\cdots]\}.\end{aligned} \tag{4}$$

For a constant beamwidth transducer we want the farfield pressure amplitude $|p_f|$ to be independent of ka over as wide a frequency range as possible. Consider what happens at ka high enough so that one can use the asymptotic form of $j_m(ka)$. Then,

1266 J. Acoust. Soc. Am. 64(5), Nov. 1978 1266

U.S. Navy CBT Paper #3 (1983)



Experimental constant beamwidth transducer

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(Received 26 October 1982; accepted for publication 11 February 1983)

The theory of a broadband constant beamwidth transducer (CBT) which is based on Legendre function shading of a spherical cap was described in a previous report [P. H. Rogers and A. L. Van Buren, J. Acoust. Soc. Am. **64**, 38–43 (1978)]. Theoretical calculations showed the CBT to have uniform acoustic loading, extremely low side lobes, virtually no nearfield, an essentially constant beam pattern for all frequencies above a certain cutoff frequency, and a flat transmitting current response over a broadband for piezoelectric drive. In this paper we present experimental results obtained for a prototype CBT based on fifth-order Legendre function shading. The transducer is a spherical cap with a diameter of 30.5 cm and a total cap angle of 50°. Measurements on the CBT showed it to possess very low side lobes and a nearly constant beamwidth over the frequency range from 20 to 140 kHz for ambient pressures up to 3.4 MPa. We also present formulas useful in the design of a CBT which is to be used as a transmitter and/or receiver. The formulas involve simple algebraic and trigonometric expressions which can be readily evaluated using a pocket calculator.

PACS numbers: 43.88.Ar, 43.30.Yj, 43.30.Jx, 43.20.Px

INTRODUCTION

Most directional acoustic transducers and arrays exhibit beam patterns which are frequency dependent. Thus, the spectral content of the transmitted or received signal varies with position in the beam, and the fidelity of an underwater acoustic system depends on the relative orientation of the transmitter and receiver. In a previous paper[1] we described a simple method for obtaining a transducer whose beamwidth is essentially independent of frequency over a broad bandwidth. Our constant beamwidth transducer (CBT) is a rigid spherical cap of arbitrary half-angle α shaded so that the normal velocity on the outer surface is proportional to $P_\nu(\cos\theta)$, where P_ν is the Legendre function whose root of smallest angle occurs at $\theta=\alpha$. That is, the normal velocity is a maximum at the center of the cap at $\theta=0$ and decreases according to $P_\nu(\cos\theta)$ until it is zero at $\theta=\alpha$. For every choice of α there is a corresponding choice for ν, the order of the Legendre function. The required value for ν is not, in general, an integer.

Theoretical calculations show that the CBT has uniform acoustic loading, extremely low side lobes, and an essentially constant beam pattern for all frequencies above a certain cutoff frequency. Under piezoelectric drive the transducer has a flat transmitting current response over a broadband. In addition, the CBT has virtually no nearfield; i.e., the acoustic pressure distribution (with respect to θ) is essentially the same at all radial distances from near the transducer surface to the farfield. The surface pressure distribution as well as the pressure distribution out to the farfield is approximately equal to the surface velocity distribution.

Although the previous paper[1] emphasized the transmitter aspects of the CBT, the concept is equally applicable to receivers. The concept is also applicable to an acoustically transparent spherical cap. In fact, Trott[2] has applied the results presented in an earlier unpublished version of Ref. 1 to the theoretical design of a receiver CBT using $P_1(\cos\theta)=\cos\theta$ shading on an acoustically transparent hemispherical cap. However, the beam pattern from such a CBT is bidirectional, containing a backlobe identical to the frontlobe.

In this paper we describe the experimental confirmation of the CBT concept. We present results obtained for a prototype CBT based on fifth-order Legendre function shading. We also present formulas useful in the design of a CBT which is to be used as a transmitter and/or receiver. The formulas involve simple algebraic and trigonometric expressions which can be readily evaluated using a pocket calculator. They are based on excellent approximations for the Legendre functions of noninteger order, and thus avoid the need for evaluating these rarely encountered functions. We obtain formulas for the required velocity distribution, cutoff frequency, directivity index, equivalent two-way beamwidths for volume and surface reverberation, and spherical cap size. The formulas are presented in Sec. I. Design considerations are discussed in Sec. II. Included are shading coefficients for a CBT with a stepwise implementation of the Legendre function shading distribution. The prototype CBT is described in Sec. III. Results of measurements on the prototype are given in Sec. IV. Last, Sec. V contains the summary and conclusions.

I. THEORY

A. Velocity distribution

The key to the special properties of the CBT is its surface normal velocity distribution $u(\theta)$ given by

$$\begin{aligned}u(\theta)&=P_\nu(\cos\theta),\quad 0<\theta<\alpha_\nu,\\u(\theta)&=0,\quad \theta>\alpha_\nu,\end{aligned} \tag{1}$$

where α_ν is the zero of smallest angle of the Legendre function $P_\nu(\cos\theta)$. We showed in Ref. 1 that the CBT need not extend beyond α_ν, and thus can be constructed as a spherical cap whose half-angle is equal to α_ν. Consequently, we call α_ν the cap half-angle, although in practice the actual cap half-angle for a P_ν CBT can possess any value $\theta>\alpha_\nu$. The order ν of the Legendre function can be chosen to be any real num-

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